HYDRAULIC TURBOMACHINES

Exercises 4

Pelton Turbines

The hydro power plant of San Carlos is located in the Central Colombian Cordillera, 50 km from Medellín. It takes water from the Nare and Guatapé rivers, which join the Magdalena river that flows to the Caribbean Sea. An earth dam 800 m long forms a lake with a volume of $72 \cdot 10^6$ m^3 from which $50 \cdot 10^6$ m^3 can be used. The water distribution structure is divided into two branches that supply 4 turbines each. The two headrace tunnels have a length of 4.5 km and a diameter of 6.1 m. Each of them is followed by a surge shaft and a penstock 600 m in length and 4.0 m in diameter. Each Pelton turbine, one of which is illustrated in Figure 1, provides a mechanical power of 170.4 MW using an available specific energy $E = 5668$ J kg^{-1} and discharge $Q = 33.40$ $m^3\,s^{-1}$. The pitch diameter of the runners is $D = 3.25$ m, the frequency of the Colombian electrical grid is 60 Hz, and the water density is equal to 999.0 kg m^{-3}

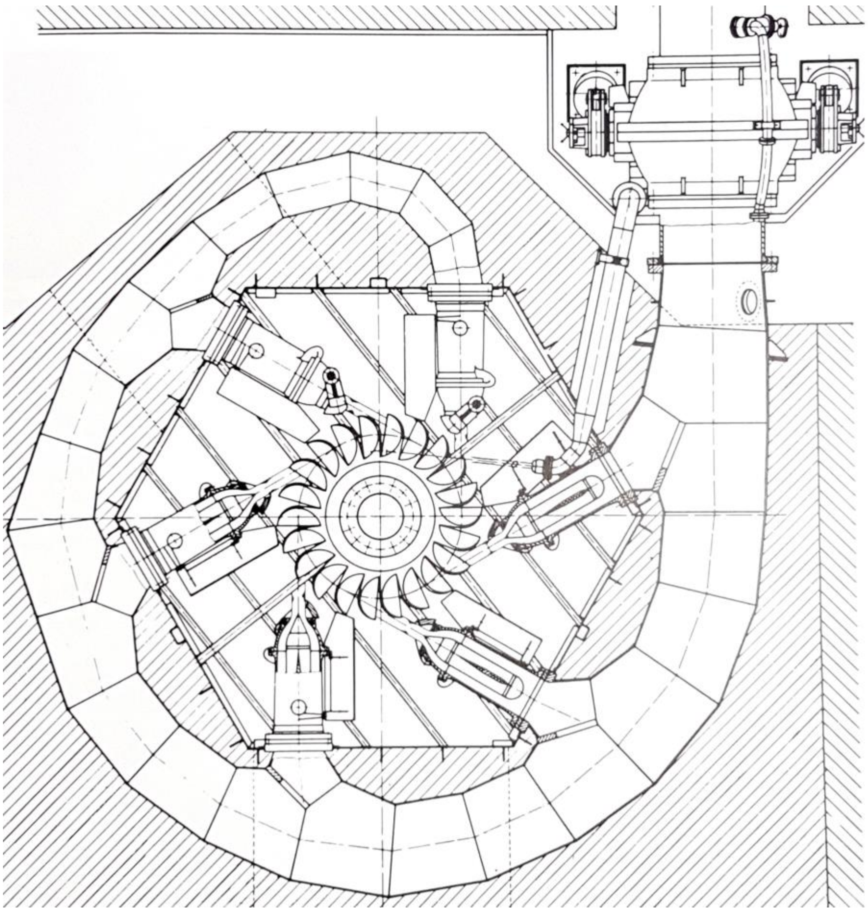

Figure 1: Sketch of a Pelton turbine from the San Carlos power plant.

1 Hydraulic System

1. Neglecting the specific kinetic energy in the upper reservoir and assuming that the pressure on its free surface and inside of the Pelton turbine housings is equal to the atmospheric pressure, show that the difference of specific energy $gH_B - gH_{\bar{I}}$ between the upper reservoir and the turbine outlet is equal to $g(Z_B - Z_{\bar{I}}) - \frac{1}{2}C_{\bar{I}}^2$.

$$gH_B - gH_{\bar{I}} = \left[\frac{p}{\rho} + gZ_B + \frac{C_B^2}{2}\right] - \left[\frac{p}{\rho} + gZ_{\bar{I}} + \frac{C_{\bar{I}}^2}{2}\right] \tag{1}$$

$$= \left[\frac{p_{atm}}{\rho} + gZ_B + 0\right] - \left[\frac{p_{atm}}{\rho} + gZ_{\bar{I}} + \frac{C_{\bar{I}}^2}{2}\right] \tag{2}$$

$$= g(Z_B - gZ_{\bar{I}}) - \frac{C_{\bar{I}}^2}{2} \tag{3}$$

2. What is the difference between $gH_B - gH_{\bar{I}}$ and the specific hydraulic energy E available to the machine? How can this difference be minimized?

   The difference corresponds to the specific energy losses in the water distribution system. It can be minimized by increasing the tunnel and penstock diameters, decreasing their length, or decreasing their roughness.

3. Similar to the first week exercise, it is possible to derive a detailed catalog of regular and singular specific energy losses throughout the hydraulic system taking into account its geometry and wall roughness properties. To avoid all this repetitive work, assume that $\sum_k gH_{r_k}$, the sum of energy losses throughout each branch of the distribution system, is equal to $K\frac{C_2^2}{2}$, where $K = 10.0$ and C_2 is the mean velocity in the penstock. Calculate the power lost throughout the hydraulic system and compare it with the power per runner. Does the computed value make sense?

$$C_2 = \frac{4Q}{\frac{\pi}{4}D_p^2} = 10.63 \text{ m s}^{-1} \tag{4}$$

$$\sum_k gH_{r_k} = K\frac{C_2^2}{2} = E_r = 565.1 \text{ J kg}^{-1} \tag{5}$$

$$P_r = \rho\ (4Q)\ E_r \times 2 = 150.9 \text{ MW} \tag{6}$$

   The power dissipated corresponds to 89% of the power provided by a single turbine. This might sound like a lot, but in fact it has to be compared to the power of the whole installation (8 turbines); it corresponds to 11% of the power provided by all the turbines. The power dissipated in the conveying system is an engineering choice and results from a trade-off between the cost of the civil works and the losses of profit due to dissipated power.

2 Pelton Runner

4. The specific energy available to the Pelton turbine is defined between the distributor (immediately upstream of the injectors) and the turbine outlet. Calculate the jet velocity

C_1 and the specific energy losses in one injector knowing that the jet diameter is $D_\circ = 0.263$ m.

$$C_1 = \frac{\frac{Q}{z_\circ}}{\frac{\pi}{4}D_\circ^2} = 102.47 \text{ m s}^{-1} \quad (7)$$

$$E_r = E - E_{jet} = E - \frac{C_1^2}{2} = 418.02 \text{ J kg}^{-1} \quad (8)$$

5. The generators at the San Carlos power plant feature 12 pole pairs. Calculate the runner peripheral velocity U_1. Does the ratio $\frac{U_1}{C_1}$ match the ideal value derived from the simplified 2D theory?

$$U = \frac{D}{2}\omega = \frac{D}{2}\frac{2\pi f}{z_p} = 51.05 \text{ m s}^{-1} \quad (9)$$

$$\frac{U}{C_1} = 0.498 \simeq 0.5 \text{ (condition to have maximum } E_t\text{)} \quad (10)$$

6. Knowing that the mean bucket outlet angle $\beta_{\bar{I}} = 12.8°$ and assuming that there are no specific energy losses between the bucket inlet and outlet (the fluid loses no energy in its path through the bucket), calculate the outlet absolute velocity $C_{\bar{I}}$ and angle $\alpha_{\bar{I}}$.

$$W_1 = W_{\bar{I}} = W = C_1 - U \quad (11)$$

$$\alpha_{\bar{I}} = \text{atan}\left(\frac{-W\sin(\beta_{\bar{I}})}{W\cos(\beta_{\bar{I}}) - U}\right) = 85.4° \quad (12)$$

$$C_{\bar{I}} = W\frac{\sin(\beta_{\bar{I}})}{\sin(\alpha_{\bar{I}})} = 11.43 \text{ m s}^{-1} \quad (13)$$

7. Properly draw the velocity triangles at the bucket inlet and outlet.

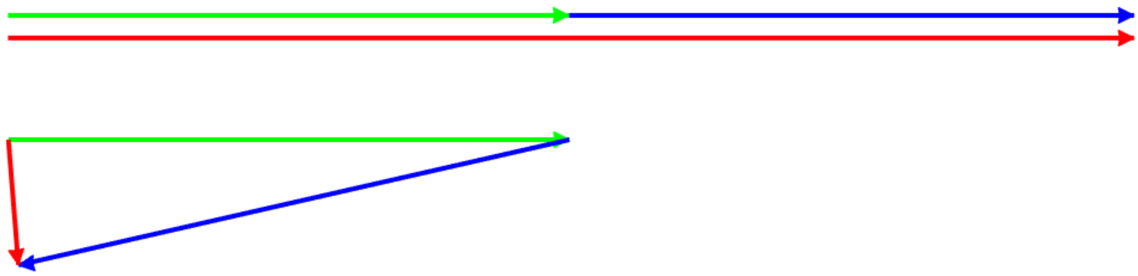

*Figure 2: Velocity triangles at the inlet (top) and outlet (bottom) of the bucket. They are composed of the absolute fluid velocity **C** (red), relative fluid velocity **W** (blue) and runner tangential velocity **U** (green).*

8. The transformed specific energy in the runner can be expressed as $E_t = U_1(C_1 - C_{\bar{I}}\cos(\alpha_{\bar{I}}))$. How will this quantity change if the assumption of no specific energy losses in the bucket is false? Sketch the resulting velocity triangle at the bucket outlet.

   If there are energy losses in the bucket, the relative velocity at the outlet $W_{\bar{I}}$ will be lower than the relative velocity at the inlet W_1, resulting in a smaller value of $\alpha_{\bar{I}}$. Therefore the transformed specific energy E_t will decrease.

9. Calculate E_t. Is the transformed power $P_t = \rho Q E_t$ equal to the mechanical power P measured at the turbine shaft? Why?

$$E_t = U_1(C_1 - C_{\bar{I}}\cos(\alpha_{\bar{I}})) = 5184.7 \text{ J kg}^{-1} \quad (14)$$

$$P_t = \rho Q E_t = 173.0\text{MW} = 1.015 \times P \quad (15)$$

   Whereas P_t is the mechanical power effectively extracted by the turbine, P is the mechanical power that is measured in the shaft. The difference is explained by the frictional losses in the shaft bearings, as well as the losses in the Pelton bucket which were neglected in the calculation of E_t (by assuming that $W_1 = W_{\bar{I}}$ in Eq. 11).